Demandable Debt and Leverage Ratchet Effect

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Motivation behind this paper

The usual setup

- Borrower making financing and default decisions w/o commitment
- Competitive lenders
- Various motives for borrower to take on debt (taxes, impatience, etc)
- Equilibrium concept: MPE (usually...)

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This paper

- focus on *demandable debt contracts*
- discrete time, and limit as $\Delta \to 0$

Short term debt in discrete time: setup

Assumption

- Time interval Δ
- Risk-neutral borrower with discount rate δ
- Competitive, risk neutral creditors with discount rate $r < \delta$
- Borrower's income rate follows Markov process
- Debt contract available: one period debt
- Upon default, entire income stream is lost
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Notation

- Debt face value F
- Borrower's income Y
- Debt price $D(Y, F)$
- Borrower's (continuation) value function $V(Y, F)$

Short term debt in discrete time: solution

Borrower's Bellman equation

$$V(Y,F) = \max_{F'} Y\Delta + D(Y,F')F' - F + e^{-\delta\Delta}\mathbb{E}_Y\left[\max\left(0, V(Y',F')\right)\right]$$

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$$\partial_F V(Y,F) = -1$$

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Resulting FOC

$$D(Y,F') + F'\partial_F D(Y,F') = e^{-\delta\Delta}\mathbb{E}_Y\left[1_{\{V(Y',F')\geq 0\}}\right] = e^{(r-\delta)\Delta}D(Y,F')$$

Short term debt in discrete time: optimal policy

⇒ Debt policy F^* satisfies

$$F^* = \frac{1 - e^{(r-\delta)\Delta}}{\frac{-\partial_F D(Y,F^*)}{D(Y,F^*)}} = \frac{\text{discount rate wedge}}{\text{bond price semi-elasticity}}$$

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Special case: i.i.d. income growth shocks $\Rightarrow$ state $x := F/Y$

- Target debt-to-income x^*
- Default when debt-to-income $x > \bar{x}$
- $x^*, \bar{x}$ solve 2 non-linear equations in 2 unknown

Short term debt in discrete time: illustration

- **i.i.d. growth** $\tilde{g} = \frac{Y'}{Y}$

$$\tilde{g} = \exp\left(\left(\mu - \frac{\sigma^2}{2}\right)\Delta + \sigma\sqrt{\Delta}\tilde{\omega}\right)$$
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  - autarky $1/(\delta - \mu)$
  - first best $1/(r - \mu)$

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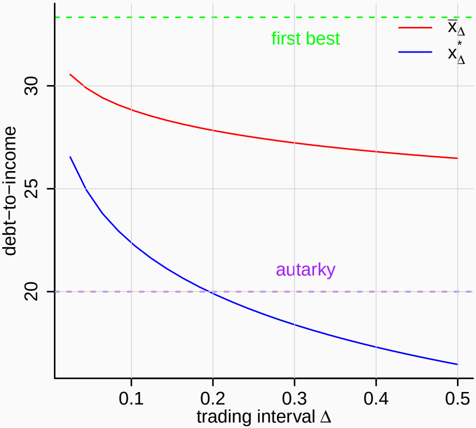

Short term debt, when $\Delta \to 0$

When income is a geometric Brownian motion

- first best achieved

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When income is a jump-diffusion: example

- $\frac{dY_t}{Y_{t-}} = \mu dt + (\tilde{\gamma}_t - 1)dN_t$
  - jump distribution $\tilde{\gamma} \sim H$
  - jump intensity λ
- value function $V(Y, F) = Yv(x)$, with $v(0) := v_0$
- optimal policy: target debt-to-income $x^* = \frac{\delta - r}{s'(x^*)}$
- bond spread $s(x) = \lambda H(x/v_0)$

Back to this paper

Debt contract

- paper talks about *demandable debt* or *demand deposit*
- not a big fan of these words
  - the contract is very far from a standard bank deposit contract
  - the debt in the model can trade at a substantial premium
- would rather use *long-term debt with a par put*

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Results

- When $\Delta > 0$, target leverage x^* and default if leverage $x > \bar{x}$ (with shock is "sufficiently large")
- When $\Delta \to 0$, $x^* = \bar{x}$, first best achieved, no defaults
- Identical to short term debt model $\Rightarrow$
  - In what way these models differ?
  - Are the critical values $x^*, \bar{x}$ identical in both models?
  - Is the economic mechanism restoring first best identical in both models

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Exposition

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- upside/simplifications in jumping directly to continuous time limit?
- analytical tractability?

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How special is the result?

- Restoration of gains from trade/first best: is this due to the special nature of the income process?
- Can paper push further and instead analyze jump processes? jump-diffusions?
- Can paper generalize to 2 state variables?